

C&RP 870.02 - DYNAMIC OPTIMIZATION IN PLANNING

Credit: 5 hours
Call No. 04109-9
Place: 274 Brown Hall
Time: To be arranged
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INTRODUCTION

The purpose of the course is to introduce students to the theory and applications of dynamic optimization. After a review of integral calculus and differential equation, the course will cover such topics as the calculus of variations and the maximum principle (or optimal control theory), with a strong emphasis on applications to management sciences, economics, regional science, and urban and regional planning.

COURSE OBJECTIVES

At the end of the course, the student should be (1) familiar with the basic elements of the calculus of variation and optimal control theory, (2) able to mathematically solve simple dynamic optimization problems, and (3) able to apply these decision-making techniques to practical problems, particularly in urban and regional planning.

COURSE METHODS

The class will be conducted in a lecture format. Homework problems will be assigned regularly, which are designed to facilitate understanding of the concepts discussed in class. There will be a final exam.

No single textbook is used for this course, as the class materials are culled and constructed from numerous articles and books in the economics, regional science, and operations research literature. However, the following textbooks will be available on reserve at the Science and Engineering Library and can be usefully consulted:

M. D. Intriligator - Mathematical Optimization and Economic Theory, Prentice-Hall, Inc. 1971.

M. I. Kamien and N. L. Schwartz - Dynamic Optimization The Calculus of Variations

and Optimal Control in Economics and Management, North-Holland, 1981.

EVALUATION

Students will be evaluated with the following weights:

Homeworks	50%
Final Exam	50%

COURSE OUTLINE

Section I: Review of Integral Calculus

- 1.1 The Indefinite Integral. Integration by Substitution, by Parts, and by Partial Fractions. Special Integration Cases.
- 1.2 The Definite Integral.
- 1.3 Consumers' and Producers' Surplus.

Section II: Review of Differential Equations

- 2.1 Classification of Differential Equations.
- 2.2 First-Order Linear Differential Equations. First-Order Nonlinear Differential Equations: Separable Equations, Exact Equations, Integrating Factors. Homogeneous Equations. Incomplete Equations, Equation of Bernoulli, Equation of Riccati.
- 2.3 First-Order Linear Systems with Constant Coefficients. Elimination and Matrix Methods.
- 2.4 Simple Applications of First-Order Differential Equations: Interest Compounding, Epidemics Spreading, Water Quality Modeling.
- 2.5 Second-Order Differential Equations. Incomplete and Linear Equations. Special Cases. Linear Systems.
- 2.6 Higher Order Linear Differential Equations with Constant Coefficients.

Section III: Discrete Approximations for Differential Equations

- 3.1 Euler Method
- 3.2 Runge-Kutta Method

Section IV: Introduction to the Control Problem

- 4.1 Description of Dynamic Systems. State and Control Variables. Equation of Motion. Continuous vs. Discrete Systems. Simple Examples.
- 4.2 Form of the Objective Function. Problems of Bolza, Lagrange, Mayer. Some Special Cases: Time Optimal Control Problem, Servomechanism Problem, Minimum Effort Problem.
- 4.3 Constraints. Final State and Terminal Time. Constraints on State and Control Variables.
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Section V: Calculus of Variations

- 5.1 Historical Background and Basic Concept.
- 5.2 Problems without Constraints. Initial and Final States Given: Euler Equation. Special Cases of Euler Equation. Economic Interpretation. Condition of Legendre and Weierstrass.
- 5.3 Problems without Constraints. Free Final State. Transversality Condition. Free Duration. Multidimensional Case.
- 5.4 Problems with Equality and Inequality Constraints.

Section VI: Applications of the Calculus of Variations

- 6.1 Monopolistic Pricing over Time
- 6.2 Optimal Economic Growth

Section VII: Optimal control Theory--The Maximum (Pontryagin) Principle

- 7.1 Problem Formulation.
- 7.2 Costate Variables, the Hamiltonian, and the Maximum Principle.
- 7.3 Economic Interpretation of the Costate Variables.
- 7.4 The Maximum Principle with Constraints.

- 7.5 The Maximum Principle and the Calculus of Variations.
- 7.6 Examples of Applications: Shortest Distance in a Plane; Minimum Time Problem; Minimum Effort Problem; Monopolistic Pricing Over Time.

Section VIII; Optimal Control Models in Management, Economics, and Regional Science

- 8.1 Production and Storage Policy.
- 8.2 Equipment Maintenance and Replacement Policy.
- 8.3 Investment Choices and the Financial Policy of the Firm.
- 8.4 Optimal Economic Growth Revisited
- 8.5 Optimal Urban Form
- 8.6 Resources and Environmental Modeling
- 8.7 Regional Allocation of Investment
- 8.8 Space-Time Models