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Theory and Methodology

Optimizing the natural gas supply mix of local distribution utilities [1]

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Abstract

A large mixed-integer linear program (MILP) and a much smaller nonlinear programming (NLP) approximation of the MILP, involving simulation and response surface estimation via regression analysis, are proposed to solve the problem of the optimal selection of natural gas supply contracts by local gas distribution utilities. Each potential supply source is characterized by several price and nonprice parameters. Weather variability is the basic stochastic factor that drives the demand for gas by various market segments. The model minimizes the total cost of gas supply and market curtailment, and thus determines the size of the interruptible market. A numerical application of the methodology illustrates the excellent quality of the NLP approximation and the importance of the trade-offs between contract characteristics. A multi-temporal extension of the modeling methodology is outlined. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

During the past 15 years, there has been almost continuous regulatory change in the US natural gas industry, leading to significant market changes. In particular, Federal Energy Regulatory Commission (FERC) Orders 436, 500 and 636 have transformed the role of the transmission pipelines from merchants, who owned the gas they transported from the production areas to their customers, to nondiscriminatory carriers. This has shifted the focus of the industry to local gas distribution companies (LDCs), which now face complex responsibilities in making gas procurement and transportation decisions. The unbundling of pipeline sales, transportation, and storage provides LDCs an opportunity to see more clearly the costs of each service, and thus

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to achieve cost savings by building a least-cost portfolio of these services. However, the sheer number of these opportunities, and their complex interactions with end-use gas demands, make such contract portfolio selection much more complex, as the LDCs must address not only cost issues, but also supply reliability, price uncertainty, and various operational factors. Comprehensive analytical tools are therefore necessary to help LDCs make these decisions. The purpose of this paper is to present such a tool, with a focus on the optimal selection of supply contracts, also known as the "optimal supply mix" problem.

The remainder of the paper is organized as follows. Section 2 expands the discussion and definition of the gas supply problem. Section 3 presents the modeling approach. Section 4 consists in a comprehensive numerical application of the methodology. Section 5 presents a multi-period extension of the methodology. Section 6 concludes the paper and outlines areas for further research.

2. The optimal gas supply mix problem

In order to clarify the gas supply mix problem, and to set the methodology proposed in this paper in proper perspective, we discuss the following topics in this section: (1) gas demand variability, (2) load balancing, (3) major characteristics of the new supply situation, (4) objectives in building a portfolio of contracts, (5) the focus of this paper, and (6) a succinct literature review. For an extended discussion of these issues, see also Duann (1991) and EIA (1994, 1996).

A fundamental characteristics of the demand for natural gas is its high variability. Over the short-term, weather is the major factor, particularly in service territories located in colder climates and with a large share of space-heating customers. Over the medium and longer terms, population size, industrial activity, and the prices of gas and alternative fuels, are of course major determinants. Variations in industrial activities where gas is an input to the production process, also have an effect on short-term variations in gas demand. An overall measure of this variability is the load factor, equal to the ratio of the average day to the peak day demands. The requirements of the residential and commercial sectors display the highest seasonality. These customers are generally classified as core (or firm) customers, who have almost no alternatives for fuel, and thus must be supplied on an uninterrupted basis. Noncore (or interruptible) customers are those who have the ability to switch easily to another fuel or to arrange gas purchases from other sources (bypassing the LDC). The classification of a given customer as core or noncore is somehow relative, as any customer can technically establish a fuel switching capacity, albeit maybe at a very high cost. For instance, any residential customer could maintain, together with a natural gas furnace, a heating oil furnace or an electric heating system. Hence, the costs of such alternatives, versus the cost of receiving natural gas, should, to a large extent, determine which market segments are core and noncore. Such a consideration is a major feature of the methodology presented in Section 3.

A basic planning problem for an LDC is that of supply-demand balance. On any given day with a given potential demand, the LDC must decide how much gas to take from its various supply sources, and how much to allocate to the various customer classes, possibly curtailing some of them if the potential demand exceeds the potential aggregate supply. This process is called load dispatching. In the absence of storage, and when supply sources are characterized only by unit commodity costs, supplies are ordered by increasing unit cost, demands are ordered by decreasing priority, and demands are assigned to supplies until one or the other runs out. However, this simple economic dispatch becomes more complicated when supply contracts are characterized by take-or-pay (T-O-P) clauses, which specify a share (e.g., 60%) of the maximum contract demand, that must be paid for, even if the gas is not taken. In such a case, supply sources are first ranked in terms of decreasing unit commodity cost, gas is taken in that order to avoid all (or at least the most expensive) T-O-P deficits, and then the remaining volume (non-T-O-P) is ordered from sources ranked by increasing unit cost. Of course, this daily dispatching takes as given the maximum daily supply available from each source. However, this maximum daily supply is a major decision variable in supply contracting, and must be selected before any daily dispatching takes place. The availability of seasonal underground storage further complicates the supply planning and dispatching problem. During the summer season, when the demand is low, gas is injected into storage; during the winter heating season, that gas is withdrawn from storage to complement the other supply sources. The use of storage is driven in opposite directions by considerations of reliability and economics. Reliability suggests high storage inventories to allow for demand peak forecasts that turn out to be low, and unexpected supply interruptions. However, gas inventory represents an investment of capital, and is subject to carrying costs. Hence, economics suggests low inventories. Also, storage introduces complex intertemporal considerations: for instance, interruptible customers can be curtailed today instead of withdrawing gas from storage to serve them. This gas can then be withdrawn later on to serve core customers. Also, the volumes of gas injection or withdrawal depend upon reservoir pressure, hence upon the inventory, hence upon past storage decisions. Finally, the dispatching problem may be constrained by operational considerations, such as the transmission capacity of the LDC-owned pipelines that carry gas to the various load centers.

Before the current restructuring of the gas industry, LDCs used to buy most of their gas from one or very few interstate pipelines under long-term contracts (10–20 y), under fixed price conditions. In this very stable environment, supply costs were simply passed on to the LDC's customers, and there was little attempt to minimize supply costs. Now that pipelines act as carriers, and no longer as gas merchants, LDCs purchase gas directly from producers or marketers, arrange for transportation services with one or more pipelines, and, more and more, contract for seasonal storage services. Supply contracts vary considerably in terms of duration and pricing and nonpricing clauses. They include:

1. Short-term contracts, where fixed daily volume deliveries, at fixed price, are arranged for one month or less; they allow for short-term unbalances in supply and/or demand to be corrected; such contracts make up what is called the spot market.
2. Mid-term contracts, for periods of up to 18 months, with variable prices indexed to some future or spot price, and with fixed reservation and service fees, irrespective of volumes taken.
3. Long-term contracts, for periods of 18 months to 15–20 y, with reservation fees and minimum take provisions; prices are indexed, and contracts often include renegotiation and market-out clauses.

In addition to contracting with gas suppliers, LDCs must also contract for firm and/or interruptible transportation capacity with the pipelines that will carry gas from the suppliers to the LDC. Such contracts are further complicated by capacity release programs, which enable an LDC with reserved transportation capacity to release excess capacity to a replacement shipper, with revenues offsetting some of the firm transportation reservation costs. Similar contracting decisions (e.g., firms vs. interruptible) can be made with regard to storage services. Finally, the LDC may use financial instruments to manage the risks in volatile gas prices (e.g., futures contracts, options, swaps).

While these new contracting opportunities offer savings potential for LDCs, they also carry significant risks. Reliance on short-term agreements may render the LDC vulnerable to both price shocks and gas shortages if, at some future time, a current excess supply disappears and long-term contracts are difficult to obtain. Short-term cost savings may then be outweighed by higher long-term costs, and reliable supply to core customers may be jeopardized. On the other hand, too cautious an approach, relying on long-term contracts only, may deny customers significant opportunities for rate decreases. Thus, the LDC must build a portfolio of contracts for gas purchases, transportation, and other services, that balances the conflicting criteria of short-term cost, long-term costs, price stability, supply reliability for core customers, and overall gas transportation availability. To build such a portfolio requires taking into account all potential contracts characteristics (durations, prices, volumes, minimum takes, etc.) and all market segments characteristics (demand variability, curtailment costs, expected growth, etc.), within the framework of a single- or multi-objective optimization model.

The purpose of this paper is to develop such a model for a simplified portfolio problem where: (1) simple supply contracts, of equal duration, are considered, characterized by a peak deliverability, a T-O-P minimum, and demand and commodity charges; (2) transportation and storage services are ignored; and (3) contract characteristics remain constant. This model might constitute the kernel of larger, more comprehensive models that would account for the other above-mentioned factors. An example of such extensions is presented in Section 5.

In view of the practical importance of the LDC supply portfolio problem, it is surprising to discover how small the literature on this subject is. O'Neill et al. (1979) develop a network model that allocates gas from supply nodes to demand nodes while accounting for mass conservation and pressure constraints across a pipeline network, but they do not account for weather-related demand variability. Guldmann (1983) analyzes the interactions between gas supply, storage, and service reliability, using chance-constrained programming and accounting for random weather variability from month to month. While storage operational constraints are very detailed, only one supplier is considered. In contrast, several suppliers and their contract characteristics are considered in Guldmann (1986), but in a deterministic framework, i.e., only monthly expected demands are considered. Also, the focus is primarily on pricing, and not on supply mix optimization. A more comprehensive approach is proposed in Avery et al. (1992), dealing with both supply and transportation/storage contracts, as well as with the network structure of the pipeline companies considered, but still in a deterministic framework. It is, however, clear that much work is being carried on this issue within the industry (e.g., Jacobs, 1993) and its consultants (e.g., Planmetrics, 1988). Unfortunately, much of this work is proprietary and remains unpublished.

3. The modeling approach

3.1. Definitions and assumptions

Consider a given market of potential end-use gas consumers served by an LDC. This market can be segmented into M $(m = 1, \ldots, M)$ homogeneous submarkets (e.g., residential, commercial, industrial, etc.). The basic factor that drives variations in submarket gas demand is the weather, as characterized by the number of daily heating degree-days X, which is defined as the difference between the reference (thermal equilibrium) average daily temperature of 65°F and the actual average daily temperature T. More precisely, $X = \max(0, 65-T)$, which implies that $X = 0$ whenever $T \geqslant 65$°F. We assume that a statistical analysis of historical daily degree-day data indicates that there are S $(s = 1, \ldots, S)$ possible different states for this random variable, and we define X_s to be the number of daily heating degree-days for state s, and P_s to be the probability of occurrence of state s.

The daily gas requirement of submarket m under state s is a linear function of X_s, with

$$Q_{ms} = A_m + B_m X_s, \quad (1)$$

where A_m and B_m are the base-load and heating-load coefficients for submarket m, respectively. Some market segments may have a constant requirement (e.g., industrial users that need gas in production processes only), thus $B_m = 0$. Other segments may use gas for space-heating only (e.g., some residential users), thus $A_m = 0$. However, in general, both requirement components are present in all submarkets, albeit in highly varying degrees. We assume that A_m and B_m are exogenously specified and remain constant within the framework of this analysis, although it is clear that they do vary over the long term as functions of the prices of gas and alternative competing energy sources, as well as of the costs of energy conservation.

Curtailment of market m occurs if the demand Q_{ms} is not fully satisfied. Such curtailment entails a cost, which may be related to a loss of production for an industrial user, or to the use of a more expensive backup source of energy (e.g., oil burning in dual-fuel furnaces), or to other forms of hardship (e.g., school closings, health effects, etc.). We assume that the total curtailment cost is proportional to the amount of gas curtailed. This constant unit cost assumption may not be fully satisfactory if market m is substantially heterogeneous, that is, made of sub-markets with different unit costs (e.g., industry). Further market segmentation would be the simplest way to solve this problem, but would increase the size of the model. We define CR_m to be the unit curtailment cost for market segment m.

To satisfy its customers, the LDC can purchase gas from $N(i = 1, \ldots, N)$ suppliers, which provide gas under specific contract characteristics. In the following, we shall use the terms "supplier", "supply source", and "contract" interchangeably. Each contract i is characterized by three exogenous parameters: CC_i is the commodity charge, i.e., cost per unit of gas actually taken from source i; CD_i the demand charge, i.e., cost per unit of maximum daily deliverability (demand costs are strictly proportional to maximum deliverability); t_i the take-or-pay (T-O-P) rate, which specifies the minimum share of the demand contract, below which gas must be paid for, whether actually taken or not.

The basic initial decision that the LDC must make with regard to contract i is the demand contract D_i, equal to the maximum daily deliverability from source i. While some contracts do allow daily purchasing exceeding D_i under some circumstances, and with a cost penalty, we assume that D_i is the upper limit of daily supply from source i on any day. The minimum purchase implied by the T-O-P rate is then t_iD_i.

3.2. The optimal supply mix as a mixed-integer linear program

In some ways, the gas supply mix problem is similar to the generation mix and power dispatching problem in the electric utility industry. However, a distinct and complicating factor is the minimum purchase, which implies some form of discontinuity in the amount of gas actually taken, and requires the use of integer 0–1 variables. Let

$$Q_s = \sum_{m=1}^{M} Q_{ms} = \sum_{m=1}^{M} (A_m + B_m X_s) \tag{2}$$

be the total market requirement for gas under state s. The maximum total amount of gas that can be purchased on any day is: $\sum_i D_i$. The total amount of gas to be curtailed under state s is then

$$Q_{Cs} = \max\left(0, Q_s - \sum_i D_i\right). \tag{3}$$

Next, we define the total amount of gas required by the market under state s *beyond* the minimum total purchase, $\sum_i t_iD_i$, with:

$$Q_{Ms} = \max\left(0, Q - \sum_i t_iD_i\right). \tag{4}$$

The supply and curtailment variables under state s are Y_{is}, equal to the amount of gas taken from source i *in addition* to the minimum take t_iD_i, and Z_{ms}, equal to the amount of gas curtailed for market segment m.

The supply constraints are:

$$Y_{is} \leqslant (1 - t_i)D_i \quad (i = 1, \ldots, N; s = 1, \ldots, S), \tag{5}$$

i.e., the incremental take is limited by the demand contract D_i; and

$$\sum_{i=1}^{N} Y_{is} + \sum_{m=1}^{M} Z_{ms} = Q_{Ms} \quad (s = 1, \ldots, S), \tag{6}$$

i.e., the sum of the total incremental take and market curtailment is equal to the total excess requirement Q_{Ms}.

The curtailment constraints are:

$$Z_{ms} \leqslant Q_{ms} \quad (m = 1, \ldots, M; s = 1, \ldots, S), \tag{7}$$

i.e., it is impossible to curtail more than the total gas requirement Q_{ms}; and

$$\sum_{m=1}^{M} Z_{ms} = Q_{Cs} \quad (s = 1, \ldots, S), \tag{8}$$

i.e., the aggregate deficit Q_{Cs} must be allocated among the market segments.

The total cost to minimize is the sum of supply and curtailment costs. Supply costs include (1) the minimum bills related to contract demand and T-O-P penalties, and (2) the expected cost of the actual gas takes beyond the minimum levels. The minimum bill (or fixed) cost is

$$\mathrm{CF} = \sum_{i=1}^{N} (\mathrm{CD}_i + \mathrm{CC}_i t_i) D_i. \tag{9}$$

The expected supply cost is

$$\mathrm{CS} = \sum_{s=1}^{S} P_s \left(\sum_{i=1}^{N} \mathrm{CC}_i Y_{is} \right). \tag{10}$$

The expected curtailment cost is

$$\mathrm{CR} = \sum_{s=1}^{S} P_s \left(\sum_{m=1}^{M} \mathrm{CR}_m Z_{ms} \right). \tag{11}$$

The optimization problem is to select the values of the decision variables $\{D_i, Y_{is}, Z_{ms} \mid i = 1, \ldots, N, s = 1, \ldots, S, m = 1, \ldots, M\}$ that minimize the total cost

$$\mathrm{CT} = \mathrm{CF} + \mathrm{CS} + \mathrm{CR} \tag{12}$$

subject to constraints (3)–(8). Note that the model can set $D_i = 0$ for all supplies if curtailment costs are much lower than supply costs, which would mean that natural gas is not competitive with alternative energy supplies or nonsupply strategies such as energy conservation. At the other end of the spectrum, the D_i's may be set high enough, when natural gas supply costs are low, to eliminate the need for any curtailment.

Constraints (3), (4) cannot be used in their present forms with standard algorithms. In order to make the determination of the variables Q_{Ms} and Q_{Cs} endogenous to the model, we transform the max functions into sets of linear constraints via the introduction of new 0–1 variables:

$$w_{Ms} = \begin{cases} 1 & \text{if } Q_{Ms} = 0, \\ 0 & \text{if } Q_{Ms} = Q_s - \sum_i t_i D_i > 0, \end{cases}$$

$$w_{Cs} = \begin{cases} 1 & \text{if } Q_{Cs} = 0, \\ 0 & \text{if } Q_{Cs} = Q_s - \sum_i D_i > 0. \end{cases}$$

The following constraints guarantee the proper determination of Q_{Ms} and Q_{Cs}, with M_0 being a large given number:

$$Q_{Ms} \leqslant (1 - w_{Ms}) M_0, \tag{13}$$

$$Q_{Ms} \geqslant Q_s - \sum_i t_i D_i, \tag{14}$$

$$Q_{Ms} \leqslant Q_s - \sum_i t_i D_i + w_{Ms} M_0, \tag{15}$$

$$Q_{Cs} \leqslant (1 - w_{Cs}) M_0, \tag{16}$$

$$Q_{Cs} \geqslant Q_s - \sum_i D_i, \tag{17}$$

$$Q_{Cs} \leqslant Q_s - \sum_i D_i + w_{Cs} M_0. \tag{18}$$

The final model, made of the objective function (12) and the constraints (5)–(8) and (13)–(18), is a mixed-integer linear program (MILP). The final set of decision variables is: $\{D_i, Y_{is}, Z_{ms}, Q_{Ms}, Q_{Cs}, w_{Ms}, w_{Cs}\}$. The total number of variables is $NV = NS + MS + N + 4S$, of which $2S$ are pure 0–1 variables. The number of constraints is $NC = (N + M + 8)S$.

It is worth noting that the model is structurally a two-stage stochastic programming problem with recourse (Wets, 1966), where the D_i's are the first-stage decision variables, and all the other variables are the second-stage or recourse variables. However, in the standard model, only one of the S scenarios does materialize, and the recourse variables are selected accordingly. In the present model, all the S scenarios do take place sometime during the year. The daily cost TC, when multiplied by 365, represents the total annual cost of supply and curtailment. Under the assumption that the weather pattern (i.e., the distribution of X_s) is the same from year to year, this total annual cost is deterministic.

The size of the MILP is highly dependent on S, the number of possible values of daily heating degree-days. If we assume that $S = 100$, the number of market segments $M = 10$, and the number of supply sources $N = 10$, then the model would include 2800 constraints, 2210 continuous variables, and 200 integer variables. A value of $S = 100$ would correspond to an area where average daily temperatures (rounded to the closest integer value) take all values between -34°F and 65°F during the year, a reasonable assumption for many northern US states. While a model of this size can be solved with available MILP algorithms, such computations might become cumbersome in the framework of an extensive sensitivity analysis over the model parameters or for much larger model sizes. Another approach would be to implement a resource directive decomposition of the MILP, using Benders' algorithm (see, for instance, Shapiro, 1979, Chapter 6, and Bienstock and Shapiro, 1988), wherein the coordination problem would involve the choice of D_i's, and a subproblem would be defined for each state s and would involve the corresponding optimal dispatch and curtailment for given D_i's.

3.3. The optical supply mix as a combined simulation/optimization

As an alternative to MILP branch-and-bound algorithm or decomposition methods, we propose a combined simulation/response curve estimation/optimization approach to solve the supply mix problem. Consider a given state measured by X_s, and assume that all the contract demands D_i $(i = 1, \dots, N)$ are given exogenously. Under such conditions, the optimal (least-cost) sequencing of gas takes from the N sources by the gas dispatcher is a straightforward task:

(1) If the total market demand Q_s, (Eq. (2)) is less than the sum of the T-O-P minimum purchases $(\sum_i t_i D_i)$, then the minimum total bill CF (Eq. (9)) applies, and there are no other costs. The dispatcher can take gas from any source i up to the minimum level $t_i D_i$ without any effect on CF.

(2) If $\sum_i t_i D_i \leqslant Q_s \leqslant \sum_i D_i$, then the dispatcher will first exhaust the minimum takes $t_i D_i$ and then will satisfy the residual gas requirement $(Q_s - \sum_i t_i D_i)$ by purchasing gas from the N sources ranked in terms of increasing commodity charge CC_i. The dispatcher starts purchasing from source i only after having exhausted the available supply from source $i-1$, that is: $Y_{is} > 0$ iff $Y_{i-1s} = (1 - t_i) D_i$. The total supply cost for state s is then: $\sum_i \mathrm{CC}_i Y_{is}$.

(3) If $Q_s > \sum_i D_i$, then curtailments are necessary. The gas dispatcher purchases all the available gas $(\sum_i D_i)$, and the total supply cost is then: $\sum_i (\mathrm{CD}_i + \mathrm{CC}_i) D_i$. The total curtailment volume, $Q_s - \sum_i D_i$, is then allocated to the various market segments in terms of increasing unit curtailment cost. The dispatcher starts curtailing segment m only if segment $m-1$ has been completely curtailed, that is: $Z_{ms} > 0$ iff $Z_{m-1s} = Q_{m-1s}$. Note, however, that the total curtailment of market m under all states, $\sum_s Z_{ms}$, would, generally, only represent a partial curtailment of its total requirement, $\sum_s Q_{ms}$. The total curtailment cost for state s is then: $\sum_m \mathrm{CR}_m Z_{ms}$. It must be added to the fixed and supply costs to obtain the total cost CT_s.

The above simulation is then repeated for all states s, leading to a set of costs CT_s $(s = 1, \dots, S)$. The next step is to weigh these costs by their probabilities, with:

$$\mathrm{CT} = \sum_{s=1}^{S} P_s \mathrm{CT}_s. \tag{19}$$

CT is the minimum total cost for *given* contract demands D_i $(i = 1, \dots, N)$. Thus:

$$\mathrm{CT} = \mathrm{CT}(D_1, \dots, D_i, \dots, D_N). \tag{20}$$

The optimization problem now is to select the D_i's that minimize $\mathrm{CT}(\mathbf{D}), \mathbf{D} = (D_1, \dots, D_i, \dots, D_N)$. As there is no closed-form expression of $\mathrm{CT}(\mathbf{D})$, the only possible approach is to approximate this function by (1) generating a response surface by varying the D_i's extensively over a grid of values and computing CT by simulation for every combination of the D_i's and (2) estimating the response surface $\mathrm{CT}(\mathbf{D})$ with curve-fitting regression techniques. While the problem

$$\underset{(D_i)}{\text{Minimize}} \quad \mathrm{CT}(D_1, \dots, D_i, \dots D_N) \tag{21}$$

is considerably simpler than the earlier MILP (N continuous variables, no constraints), one could argue that this simplification does not justify the necessary large number of simulations and the regression curve-fitting. However, the approach can be expanded to include, among the arguments of CT(.), not only the decision variables $\mathbf{D}$, but also the various parameters of the model. Consider the following vectors of parameters: $\mathbf{CC} = \{\mathrm{CC}_i\}$, $\mathbf{CD} = \{\mathrm{CD}_i\}$, $\mathbf{CR} = \{\mathrm{CR}_m\}$, $\mathbf{T} = \{t_i\}$, $\mathbf{A} = \{A_m\}$, $\mathbf{B} = \{B_m\}$. By varying these parameters, together with $\mathbf{D} = \{D_i\}$, over grids of values, we could estimate a more general function

$$\mathrm{CT} = \mathrm{CT}(\mathbf{D}, \mathbf{CC}, \mathbf{CR}, \mathbf{T}, \mathbf{A}, \mathbf{B}), \tag{22}$$

which would provide a convenient approach to sensitivity analyses over the parameters. Such analyses may be critical to gas supply planners when the parameters are uncertain or may be random variables. For instance, the base and heating load coefficients **A** and **B** may change in the future due to energy conservation and/or competition from other energy sources. Analyzing the impacts of such possible changes on the optimal demand contracts **D** may help the supply planner select a robust contract portfolio. Similar arguments may be made with regard to the other parameters. Of course, the number of simulation runs would increase with an increase in the number of parameters, and, as a result, the accurate estimation of the function (22) would become more difficult. However, as illustrated in the application presented in Section 4.2, this difficulty may be alleviated by a pre-estimation analysis of the specific roles of the parameters in the different cost components.

4. Application

4.1. Data

The data used to illustrate the modeling approach are, in part, hypothetical, yet realistic, and, in part, based on actual data characterizing an LDC, the National Fuel Gas Distribution Company (NFGDC), which serves primarily the western parts of the states of New York and Pennsylvania. The availability of usable data was the primary criterion in selecting this LDC.

Gas requirement data were drawn from the Uniform Statistical Report (USR) filed by NFGDC with the American Gas Association (AGA). This report provides monthly gas sales to various customer classes (sectors), and the corresponding numbers of monthly heating degree-days. NFGDC market is segmented into four sectors: residential, commercial, industrial, and public authorities (noncommercial institutional buildings). Using the 1982 USR, we have divided each monthly gas volume and the number of degree-days by the number of days in that month. Next, we have regressed, for each sector, the average daily gas requirement on the average daily number of degree-days. The regression results, with the corresponding R^2 and t-values, are presented in Appendix A. The base and heating load coefficients (A_m and B_m) to be used here are based on these estimated values, with some rounding, and are presented in Table 1.

Daily heating degree-day data were available for Buffalo, NY (located in NFGDC service territory) for 4 yr (1976–1979). A frequency analysis of these data shows that X_s ranges from 0 to 70, with 70 distinct values (i.e., states), the only value missing in the range [0,70] is 69. The average daily heating degree-day is $\overline{X} = 19.26$. About 25% of the days of the typical year (here assumed to be the average of 1976–1979) have zero heating degree-days, and thus no space-heating requirements. Using the selected load coefficients, we have computed the average and peak daily requirements for each sector (with $\overline{X} = 19.26$ and $X_p = 70$). Their ratio, the load factor, is a measure of the variability of the daily requirements (e.g., a 100% load factor

Table 1
Gas markets load and curtailment cost data

Market segment	Load coefficients (MCF) [a]		Average load (MCF)	Peak load (MCF)	Load factor	Curtailment unit cost ($/MCF)
	Base	Space-heating				
Residential	55,000	11,000	266,838	825,000	0.323	12.00
Commercial	15,000	3000	72,774	225,000	0.323	8.00
Industrial	100,000	3000	157,774	310,000	0.509	6.00
Public authorities	5000	1200	28,110	89,000	0.316	9.00
Total	175,000	18,200	525,496	1,449,000	0.363	–

[a] MCF: thousand cubic feet.

indicates a completely constant demand). These loads and load factors are also presented in Table 1.

We consider five hypothetical gas suppliers, ranked $(1, \ldots, 5)$ in terms of their increasing commodity charge: \$2.00, \$2.50, \$3.00, \$3.50, \$4.00, per MCF (thousand cubic feet). To these charges (CC_i), which are assumed constant throughout the analysis, are associated demand charges (CD_i) and T-O-P rates (t_i). We assume that the demand charge may vary between \$0.20/MCF and \$0.80/MCF, and the T-O-P rate between 0.40 and 0.80. A primary goal of the analysis reported in Section 4.3 will be to uncover the trade-offs between these contract parameters.

The selection of the sectoral unit curtailment costs was based, in part, on the end-use prices of oil products in 1990 in the state of New York, as drawn from EIA (1992). The average price of distillate oil in the industrial sector was \$6.20 per MMBtu (Million British Thermal Units). In the commercial and residential sectors, we have computed the average of the prices of distillate fuel and liquefied petroleum gas (LPG), with \$8.50/MMBtu and \$11.00/MMBtu for commercial and residential end-users, respectively. The selected curtailment costs, presented in Table 1, are close to these values. No data were available for the Public Authorities sector, for which we selected a unit cost of \$9.00/MCF, close to but higher than the commercial unit cost.

4.2. Response curve estimation

The average daily cost CT is the sum of three components: (1) the fixed cost, CF, equal to the sum of the demand and T-O-P costs, (2) the expected supply cost, CS, and (3) the expected curtailment cost, CR. Eq. (9) in Section 3.2 provides an explicit formulation of CF, allowing the estimation process to focus on CS and CR. Consider first the case of CR. As the unit curtailment costs CR_m are fixed, it is clear that the distribution of the total daily volumes of gas curtailed, and thus the distribution of the resulting costs, CR_s, only depend upon the aggregate contract demand $D_T = \sum_i D_i$, but not upon the individual demands D_i's. Hence, a general formulation of the expected curtailment cost function is

$$\mathrm{CR} = f(D_T). \tag{23}$$

We have simulated the curtailment process by unit-incrementing D_T from 0 to 1500, and for each value of D_T we have computed CR. We have next used a third-order polynomial

$$\mathrm{CR} = a_0 + a_1 D_T + a_2 D_T^2 + a_3 D_T^3 \tag{24}$$

to approximate the true cost function (23). An excellent fit was obtained, with $R^2 = 0.999$. Also, the coefficient a_0 was estimated at 9.365, which is very close to the true average curtailment cost of 9.484 when $D_T = 0$. The case of the supply cost CS is, however, more complicated. We define the minimum take for contract i as: $Z_i = t_i D_i$. We know that supply dispatching takes place whenever $\sum_i Z_i < Q_s$, and that the total amount dispatched is limited by $D_T = \sum_i D_i$. The amount of gas dispatched from each source i will depend upon the ranges $[Z_i - D_i]$ and the commodity costs CC_i. Depending upon the state s, this amount varies between 0 and $(D_i - Z_i)$, and the corresponding cost between 0 and $CC_i(D_i - Z_i)$. However, the distribution of the supply cost for contract i clearly depends upon the values of D_j and Z_j for all the other contracts j, hence a complex interaction between the variables D_k and Z_k and the cost parameters CC_k $(k = 1, \ldots, 5)$. As the CC_k's are assumed fixed, we focus on the variations of D_k and t_k, and hence on those of Z_k. Let $\mathbf{Z} = (Z_1, \ldots, Z_5)$ and $\mathbf{D} = (D_1, \ldots, D_5)$. We can then posit a general cost relationship

$$\mathrm{CS} = g(\mathbf{Z}, \mathbf{D}). \tag{25}$$

We have simulated the dispatching model described in Section 3.3 over a grid of values for (1) the demand contracts D_i, and (2) the T-O-P rates t_i $(i = 1, \ldots, 5)$. Each D_i may take six values (0, 300, 600, 900, 1200, and 1500 MMCF), but all combinations such that $\sum_i D_i > 1500$ were discarded, because of their irrelevance with a peak daily load of 1449 MMCF (see Table 1). Each t_i may take the following values: 0.4, 0.5, 0.6, 0.7, 0.8. The total number of combinations in the input space $(\mathbf{D},\mathbf{T})$ or $(\mathbf{D},\mathbf{Z})$ is 787,500. We have next used a third-order polynomial

$$\begin{aligned}\mathrm{CS} = \alpha_0 + \sum_i \alpha_i^1 Z_i + \sum_{i,j} \alpha_{ij}^2 Z_i Z_j \\ + \sum_{i,j,k} \alpha_{ijk}^3 Z_i Z_j Z_k + \sum_i \beta_i^1 D_i \\ + \sum_{i,j} \beta_{ij}^2 D_i D_j + \sum_{i,j,k} \beta_{ijk}^3 D_i D_j D_k \qquad (26)\end{aligned}$$

to approximate the true cost function (25). An excellent fit was obtained, with $R^2 = 0.994$.

Prior to the estimations of Eqs. (24) and (26), the costs CR and CS have been divided by the average total daily gas requirement $\mathrm{TD} = \sum_s P_s Q_s$, or 525,496 MCF (see Table 1). The average daily cost AC (= CT/TD) has, as arguments, the vectors $\mathbf{D}, \mathbf{CD}$, and $\mathbf{T}$, and is summarized as follows:

$$\begin{aligned}\mathrm{AC} = (a_0 + \alpha_0) + \sum_{i=1}^{5} [(\mathrm{CD}_i + \mathrm{CC}_i t_i)/\mathrm{TED} \\ + a_1 + \alpha_i^1 t_i + \beta_i^1] D_i + \sum_{i,j} (\alpha_{ij}^2 t_i t_j + \beta_{ij}^2) D_i D_j \\ + a_2 \left(\sum_i D_i\right)^2 + \sum_{i,j,k} \left(\alpha_{ijk}^3 t_i t_j t_k + \beta_{ijk}^3\right) D_i D_j D_k \\ + a_3 \left(\sum_i D_i\right)^3. \qquad (27)\end{aligned}$$

4.3. Optimal contract selection

The MILP presented in Section 3.2 includes 1190 constraints, 140 integer 0–1 variables, and 915 continuous variables, when applied with the data presented in Section 4.1 $(S = 70, N = 5, M = 4)$. The nonlinear program (NLP) involved in minimizing the cost function (27) involves only five continuous variables (D_i's). The MILP and NLP have been solved using the OSL and MINOS solvers of GAMS (General Algebraic Modeling System). For further details, see Brooke et al. (1996). In the following, we first assess the quality of the NLP approximation by comparing its solution to the true optimum generated by the MILP. Next, we analyze under what parameter values do contracts 2–5 become competitive with contract 1.

4.3.1. Assessment of the NLP approximation

We assume here that the parameters CD_i and T_i are the same for all five contracts. Contract 1 (with CC= \$2.00/MCF) is then the only one selected. Of course, such a situation could not take place in reality, because suppliers 2–5 would always lose, and thus be out of business. It is simply a convenient way to analyze the case of one sole supplier and its contract. We vary T_i from 0.4 to 0.8 by increments of 0.1, and CD_i from 0.2 to 0.8 by increments of 0.1. The resulting minimum average costs and optimal demand contracts for supplier 1, as produced by the MILP, are presented in Tables 2 and 3. As could be expected the minimum average cost AC decreases and the demand contract D_1 increases with decreasing T-O-P rates and decreasing demand charges. The lowest AC = 2.781 ($T_i = 0.4$ and $\mathrm{CD}_i = 0.2$) represents about 65% of the highest AC = 4.308, and the lowest $D_1 = 684.6$ ($T_i = \mathrm{CD}_i = 0.8$) represents about 70% of the highest $D_1 = 971.8$ ($T_i = 0.4$ and $\mathrm{CD}_i = 0.2$), clearly underlining the strong impact that the contract parameters **T** and **CD** have on the overall supply cost and quantity. As the average daily load is 525.5 MMCF, and the peak one 1449 MMCF, it is clear that the optimal values of D_1 are always larger than the average load, and always smaller than the peak one. Thus, curtailments occur in all cases, as it is never optimal to contract for the peak load. The LDC's customer set must include interruptible customers, which will be curtailed when the actual load exceeds the contract demand. While the desirability of having interruptible customers is well known and well accepted by LDCs, the optimal mix of firm and interruptible loads is generally not easy to determine. The present model does just that.

The minimum average costs and optimal demand contracts for supplier 1, as produced by the NLP, are presented in Tables 4 and 5, together with the percentage deviations from the exact MILP solutions. The relative cost deviation varies between −1.99% (NLP underestimation) to +0.95% (NLP overestimation). In absolute terms, the cost deviation varies between 0.41% and 1.99%, with a mean of 1.33%. The relative demand deviation varies between −1.55% and 3.43%, and, in absolute terms, between 0.017% and 3.43%, with a mean of

Table 2
MILP minimum average costs ($/MCF) – contract #1

Demand charge ($MCF)	Take-or-pay rate				
	0.400	0.500	0.600	0.700	0.800
0.2	2.781	2.943	3.115	3.292	3.474
0.3	2.960	3.113	3.276	3.445	3.620
0.4	3.133	3.278	3.433	3.595	3.763
0.5	3.301	3.438	3.585	3.742	3.904
0.6	3.463	3.593	3.735	3.885	4.041
0.7	3.621	3.745	3.881	4.026	4.176
0.8	3.774	3.893	4.024	4.163	4.308

Table 3
MILP optimal contract demand (MMCF) – contract #1

Demand charge ($/MCF)	Take-or-pay rate				
	0.400	0.500	0.600	0.700	0.800
0.2	971.80	903.00	865.40	812.00	775.60
0.3	921.20	880.60	835.00	793.80	757.40
0.4	895.80	850.20	812.00	775.60	742.00
0.5	866.60	830.20	793.80	757.40	728.60
0.6	847.00	812.00	775.60	743.80	719.25
0.7	812.00	789.40	757.40	728.60	702.80
0.8	793.80	774.20	739.20	718.00	684.60

1.06%. These results suggest that the NLP solution is, generally, a very close approximation of the exact MILP solution. Using the NLP approximation is further warranted by the difference in computation time. The average time for solving the MILP on a Pentium-based 100 MHz personal computer is 54.12 CPU seconds, while the time for the NLP is 0.133 CPU seconds, implying that solving the NLP is 407 times faster than solving the MILP. While this ratio cannot be extrapolated as such to larger problems, it suggests that large computational gains may be achieved by using the simulation/estimation/NLP approach.

4.3.2. Contract trade-off analysis

Because of its relatively small size, we continue using the MILP formulation of the model to assess under what conditions contracts 2–5 become competitive with or preferable to contract 1. Throughout, we keep contract 1 parameters constant, with $T_1 = 0.8$ and $CD_1 = 0.8$ (the most unfavorable conditions). We first vary the parameters T_2 and CD_2 of contract 2 over the previous ranges, while keeping the parameters for contracts 3–5 at the same levels as contract 1. Hence contracts 3–5 cannot become active, and the choice is between contracts 1 and 2. The results are presented in Tables 6 and 7. The indifference curve for the two contracts, that is, the set of combinations (CD_2, T_2) for which the two contracts are equally attractive, is clearly suggested by Tables 6 and 7 and represents, roughly, a sub-diagonal of the tables. For any combination (CD_2, T_2) below this curve, contract 1 is always selected. Contract 2 is always selected for any combination (CD_2, T_2) above the curve. Note that both contracts are active $(D_1 > 0, D_2 > 0)$ in six cases (see footnote of Table 7).

Next, we carry the same sensitivity analysis over the parameters of contract 3 (CD_3, T_3), leaving the parameters of all other contracts at $CD_i = 0.8$ and $T_i = 0.8$. The choice is thus here between contracts 1 and 3. The results are presented in Table 8. As could be expected, lower values for CD_3 and T_3

Table 4
NLP minimum average costs ($/MCF) and deviations (%) from the MILP optimum – contract #1

Demand charge ($/MCF)	Take-or-pay rate				
	0.400	0.500	0.600	0.700	0.800
0.2	2.800	2.916	3.062	3.228	3.405
	(0.68) [a]	(−0.92)	(−1.70)	(−1.94)	(−1.99)
0.3	2.979	3.085	3.222	3.379	3.549
	(0.64)	(−0.90)	(−1.65)	(−1.92)	(−1.96)
0.4	3.154	3.251	3.379	3.528	3.691
	(0.67)	(−0.82)	(−1.57)	(−1.86)	(−1.91)
0.5	3.324	3.413	3.533	3.674	3.829
	(0.70)	(−0.73)	(−1.45)	(−1.82)	(−1.92)
0.6	3.490	3.571	3.683	3.817	3.965
	(0.78)	(−0.61)	(−1.39)	(−1.75)	(−1.88)
0.7	3.652	3.726	3.831	3.958	4.099
	(0.86)	(−0.51)	(−1.29)	(−1.69)	(−1.84)
0.8	3.810	3.877	3.975	4.095	4.230
	(0.95)	(−0.41)	(−1.22)	(−1.63)	(−1.81)

[a] The percent deviation of the NLP value from the MILP value is indicated in parentheses.

Table 5
NLP optimal contract demand (MMCF) and deviations (%) from the MILP optimum – contract #1

Demand charge ($/MCF)	Take-or-pay rate				
	0.400	0.500	0.600	0.700	0.800
0.2	957.57	902.85	852.39	806.10	763.60
	(−1.46) [a]	(−0.02)	(−1.50)	(−0.73)	(−1.55)
0.3	930.45	880.94	833.90	789.99	749.24
	(1.00)	(0.04)	(−0.13)	(−0.48)	(−1.08)
0.4	905.52	860.26	816.20	774.45	735.31
	(1.08)	(1.18)	(0.52)	(−0.15)	(−0.90)
0.5	882.31	840.63	799.19	759.40	721.76
	(1.81)	(1.26)	(0.68)	(0.26)	(−0.94)
0.6	860.51	821.89	782.80	744.81	708.57
	(1.59)	(1.22)	(0.93)	(0.14)	(−1.49)
0.7	839.89	803.94	766.97	730.63	695.71
	(3.43)	(1.84)	(1.26)	(0.28)	(−1.01)
0.8	820.29	786.69	751.63	716.84	683.16
	(3.34)	(1.61)	(1.68)	(−0.16)	(−0.21)

[a] The percent deviation of the NLP value from the MILP value is indicated in parentheses.

Table 6
MILP minimum average costs ($/MCF) – contract #2

Demand charge ($/MCF)	Take-or-pay rate				
	0.400	0.500	0.600	0.700	0.800
0.2	3.323	3.505	3.695	3.891	4.089
0.3	3.492	3.664	3.846	4.033	4.225
0.4	3.656	3.819	3.993	4.173	– [a]
0.5	3.815	3.969	4.136	4.303	–
0.6	3.968	4.117	4.274	–	–
0.7	4.119	4.252	–	–	–
0.8	4.238	4.307	–	–	–

[a] Benchmark solution (AC = 4.308, $D_1 = 684.60$, $D_2 = 0$).

Table 7
MILP optimal contract demand (MMCF) – contract #2

Demand charge ($/MCF)	Take-or-pay rate				
	0.400	0.500	0.600	0.700	0.800
0.2	903.00	848.40	804.60	757.40	719.25
0.3	877.40	823.20	777.00	739.20	702.80
0.4	848.40	804.60	759.00	721.00	0 [a]
0.5	819.80	786.80	743.80	348.64 [b]	0
0.6	793.80	759.00	553.00 [c]	0	0
0.7	758.33 [e]	546.00 [d]	0	0	0
0.8	485.33 [f]	28.93 [g]	0	0	0

[a] $D_1 = 684.60$.
[b] $D_1 = 349.53$.
[c] $D_1 = 168.00$.
[d] $D_1 = 17.27$.
[e] $D_1 = 193.20$.
[f] $D_1 = 253.87$.
[g] $D_1 = 655.67$.

Table 8
MILP minimum average cost and optimal contract demand – contract #3

Demand charge ($/MCF)	Minimum average cost ($/MCF)			Contract demand of supplier 3 (MMCF)		
	T-O-P rate			T-O-P rate		
	0.400	0.500	0.600	0.400	0.500	0.600
0.2	3.848	4.044	4.247	850.20	793.80	648.50 [b]
0.3	4.008	4.192	–	830.20	692.53 [c]	0 [a]
0.4	4.152	4.286	–	640.80 [d]	291.20 [e]	0
0.5	4.250	–	–	371.67 [f]	0 [a]	0
0.6	4.300	–	–	121.53 [g]	0	0

[a] Benchmark solution (AC = 4.308, $D_1 = 684.60$).
[b] $D_1 = 95.30$.
[c] $D_1 = 81.67$.
[d] $D_1 = 148.60$.
[e] $D_1 = 429.80$.
[f] $D_1 = 372.13$.
[g] $D_1 = 581.47$.

than in the case of contract 2 are necessary to make contract 3 competitive with contract 1, because these lower values must compensate for a higher commodity charge ($CC_3 = \$3.00/\text{MCF}$ vs .$CC_2 = \$2.50/\text{MCF}$). The sub-diagonal indifference curve moves therefore upward and leftward. Whenever $CD_3 \geqslant 0.7$ or $T_3 \geqslant 0.7$, contract 3 is never competitive. Also, note that both contracts are active ($D_1 > 0, D_3 > 0$) in six cases (see footnote of Table 8).

Similar sensitivity analyses are then carried for contracts 4 and 5. Contract 5 is never competitive with contract 1. Contract 4 is selected in only two cases ($CD_4 = 0.2, 0.3; T_4 = 0.4$), where both contracts are active ($D_1 > 0, D_4 > 0$). When $CD_4 = 0.2, AC = 4.256, D_1 = 408.10$, and $D_4 = 349.30$; when $CD_4 = 0.3$, $AC = 4.300, D_1 = 578.93$, and $D_4 = 127.67$.

5. Multi-period extension

In order to illustrate the potential of the combined simulation/optimization approach, we outline here how it could be used to deal with contracts of unequal durations and with characteristics that vary over time. Consider a time horizon made of T periods ($t = 1, \ldots, T$), usually years, and a set of N contracts ($i = 1, \ldots, N$) that may be selected at various times over this horizon. Let t_{1i} and t_{2i} be the first and last periods of the time interval over which contract i can be active. Let $\delta_{it} = 1$ if contract i can be active at t, $\delta_{it} = 0$ if not ($\delta_{it} = 1$ if $t \in [t_{1i} - t_{2i}]$). Let D_i be the contract demand decision variable, which we assume constant throughout the contract period. The decision variable vector for period t can then be defined as: $\mathbf{D}_t = (\delta_{1t}D_1, \ldots, \delta_{it}D_i, \ldots, \delta_{Nt}D_N)$. Next, consider the following vectors of parameters: $\mathrm{CC}_t = \{\delta_{it}\ \mathrm{CC}_{it}\}, \mathrm{CD}_t = \{\delta_{it}\ \mathrm{CD}_{it}\}, \mathrm{TOP}_t = \{\delta_{it}\ \mathrm{TOP}_{it}\}$, where CC_{it}, CD_{it}, and TOP_{it} are the commodity, demand, and take-or-pay rates for contract i in period t. If r is the interest rate, then the supply mix problem is to find the values of the variables D_i that minimize the present value of all supply and curtailment costs over the period $[1 - T]$, with:

$$\mathrm{Min} \quad \sum_{t=1}^{T} [\mathrm{CT}_t(\mathrm{D}_t, \mathrm{CC}_t, \mathrm{CD}_t, \mathrm{TOP}_t)/(1+r)^t]. \quad (28)$$

For each period t, the dispatch simulation would be implemented while considering only those contracts that could be active at t. Hence, a different cost function CT_t would be estimated for each period t. The above model requires, as inputs, forecasts of the contract characteristics $\mathrm{CC}_{it}, \mathrm{CD}_{it}$, and TOP_{it}. Various scenarios for these parameters could be easily considered in the above model, extending to a multi-period framework the sensitivity analyses presented in Section 4.3.

6. Conclusions and areas for further research

We have presented a modeling methodology designed to help LDCs select the optimal mix of supply contracts, all characterized by different commodity and demand charges, as well as different take-or-pay rates. Weather variability is the basic stochastic factor that drives the demand for gas by the various market segments. The model minimizes the total cost of gas supply and market curtailment, and thus determines the optimal sharing of the total demand between firm and interruptible customers. The methodology is illustrated, numerically, with actual and synthetic data. The application underlines the critical trade-offs between contract characteristics, and demonstrates that the solution of the combined simulation, response curve estimation, and simplified optimization approach is a very good approximation of the exact solution produced by a large mixed-integer linear program. An extension of the approach to a multi-period framework with variable contract characteristics and durations has been outlined. Further extensions of the methodology might involve storage capacity and injections/withdrawals decisions. However, as demonstrated in Guldmann (1983), this would require introducing the intra-annual time dimension into the model to account for the intertemporal interactions of storage operating decisions, and for time-linked stochastic weather patterns. This time dimension would allow for seasonally changing prices. The access to transportation also needs to be considered. There are clearly interactions between the selection of transportation contracts (which pipelines, whether firm or interruptible) and the selection of the supply contracts. The secondary market for capacity resale, as well as the impacts of operating constraints (the physical configuration of the LDC network often imposes limitations on its ability to take possession of gas from suppliers and deliver it to customers) should also be considered in this extended framework.

Appendix A. Daily gas requirements functions for NFGDC

Residential sector:

$$Q_R = 54,739 + 11,103\mathrm{HDD}, \qquad R^2 = 0.99$$
$$(6.93) \qquad (36.26)$$

Commercial sector:

$$Q_C = 14,978 + 2,841\mathrm{HDD}, \qquad R^2 = 0.99$$
$$(6.46) \qquad (31.61)$$

Industrial sector:

$$Q_I = 98,209 + 2,851\text{HDD}, \qquad R^2 = 0.95$$
$$(17.76) \quad (13.30)$$

Public authorities sector:

$$Q_P = 4,403 + 1,218\text{HDD}, \qquad R^2 = 0.99$$
$$(6.09) \quad (43.46)$$

Here, HDD = daily heating degree-days, $Q_{.}$ = daily gas requirement (MCF), with *t*-values in parentheses.

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